When $m=l$, $n=a$, $F=2\pi a\rho k\left\{\frac{1}{\sqrt{l^2+a^2}}-\frac{1}{a}\right\}$.

When $m=0$, $n=a$, $F=-2\pi a\rho k\left\{\frac{1}{\sqrt{l^2+a^2}}-\frac{1}{a}\right\}$.

When $n=2a$ the particle is on the surface of the cylinder,

$$\text{then } b^2=\frac{4a^2}{m^2+4a^2},\quad c^2=\frac{4a^2}{(l-m)^2+4a^2},\ d=1.$$

$\therefore$ The elliptic function of the third order in Y disappears.

PROBLEMS.

42. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Find the time of vibration of a particle *slightly* displaced from the center of a solid cylinder in direction of the axis, the matter of the cylinder attracting according to the laws of nature.

43. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

Two weights P and Q rest on the concave side of a parabola whose axis is horizontal, and are connected by a string, length l, which passes over a smooth peg at the focus, F. [*Bowser's Analytic Mechanics, page 54.*]

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

42. Proposed by W. B. ESCOTT, 6123 Ellis Avenue, Chicago, Illinois.

In a parallelogram, sides a and b, diagonals c and d, $2a^2+2b^2=c^2+d^2$. Find all the parallelograms, not rectangles, whose sides and diagonals are rational.

Examples:

a	b	c	d
4	7	9	7
16	7	21	13
8	9	13	11
8	11	17	9

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

By means of the sides and diagonals we can form, in each parallelogram, two different triangles, the sides of one being a, b, and c, and of the other, a, b, and d.

Take the triangle, sides a, b, and c and put $a=n$, $b=n+p$, and $c=2n\pm q$. From the relations of the sum and the difference of any two sides to the third side, we have the following conditions: $p\mp q>0$ and $p\mp q<2n$. For $p-q$, $c=2n+q$; and for $p+q$, $c=2n-q$.

The *median* upon c is $1/2\sqrt{2(a^2+b^2)-c^2}$. But as the diagonals of a parallelogram bisect each other, this median equals $\frac{1}{2}d$. Whence $d^2=2(a^2+b^2)-c^2=4n(p\mp q)+2p^2-q^2$.

Then $n=\dfrac{d^2-2p^2+q}{4(p\mp q)}$. But we have found that $2n>p\mp q$. Therefore

$\dfrac{d^2-2p^2+q^2}{2(p\mp q)}>p\mp q$. Whence $d>2p\mp q$.

Put $d=2p\mp q+t$. Then $a=n=\dfrac{(2p\mp q+t)^2-2p^2+q^2}{4(p\mp q)}$;

$$b=n+p=\frac{(2p\mp q+t)^2+(2p\mp q)^2-2p^2}{4(p\mp q)};$$

$$\text{and } c=2n\pm q=\frac{(2p\mp q+t)^2-(p\mp q)^2-p^2}{2(p\mp q)},$$

in which p, q, and t are any integers. p and q may also be *zero*, but only one of them in the same operation. When $p=q$ and when $q>p$, we use q only as *positive*, $[+q]$; but when $p>q$, we can use q as both *positive* and *negative*.

When numerical values, assigned to p, q, and t, render a and b or a, b, and c fractional, integral results are obtained by multiplying a, b, c, and d by the least common denominator of the fractions.

Examples:—(1). Put $p=2$, $q=1$, and $t=2$. Then, for $p+q$, $a=7/2$, $b=11/2$, $c=6$, and $d=7$; or in integers, 7, 11, 12, and 14.

(2). Put $p=3$, $q=1$, and $t=2$. Then $a=4$, $b=7$, $c=7$, and $d=9$. Also $a=4$, $b=7$, $c=9$, and $d=7$.

For $p-q$, $a=9/2$, $b=13/2$, $c=10$, $d=5$; or in integers, 9, 13, 20 and 10.

When $q=0$, or when $c=2a$, we have $a=[(2p+t)^2-2p^2]/4p$, $b=[(2p+t)^2+2p^2]/4p$, $c=[(2p+t)^2-2p^2]/2p$, and $d=2p+t$.

Examples:—(1). Put $p=1$, and $t=2$. Then $a=7/2$, $b=9/2$, $c=7$, and $d=4$; or in integers, 7, 9, 14, and 8.

(2). Put $p=t=2$. Then $a=7/2$, $b=11/2$, $c=7$, and $d=6$; or in integers, 7, 11, 14, and 12.

When $p=0$, or when $a=b$, we have $a=[(t\mp q)^2+q^2]/4q=b$,

$c=[(t\mp q)^2-q^2]/2q$, and $d=t\mp q$; or, in integral form, $a=b=(t\mp q)^2+q^2$, $c=2t(t\mp 2q)$, and $d=4q(t\mp q)$.

Examples:—(1). Put $t=q=1$. Then $a=b=5$, $c=6$, and $d=8$.

(2). Put $t=3$ and $q=1$. Then $a=b=17$, $c=30$, and $d=16$. Also $a=b=5$, $c=6$, and $d=8$.

When $q=p$, we have, $a=[(3p+t)^2-p^2]/8p$, $b=[(3p+t)^2+7p^2]/8p$, $c=[(3p+t)^2-5p^2]/4p$, and $d=3p+t$.

When $t=q=p$, we have, in integral form, $a=15p$, $b=23p$, $c=22p$, and $d=32p$.

Thus we continue making general values for a, b, c, and d, under a number of other conditions; as, $t=q$; $t=p$; $t=2q=2p$, etc.

43. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find the series of integral numbers in which the sum of any two consecutive terms is the square of their difference.

I. Solution by J. H. DRUMMOND, LL. D., Portland, Maine, and the PROPOSER.

Let x and $x+m$ be two consecutive numbers. Then we have $2x+m=m^2$, and $x=m(m-1)/2$, and $x+m=m(m+1)/2$. But $m(m+1)/2$ is the sum of the terms in the series $1+2+3+4$ ....$+m$. Hence the m^{th} term of the series required is the sum of m terms of this series, and we have 1, 3, 6, 10, 15, ...$m(m-1)/2$.

II. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee; O. W. ANTHONY, M. Sc., Professor of Mathematics, New Windsor College, New Windsor, Maryland; and BENJ. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, Ohio.

By the conditions we must have $x+y=(x-y)^2$, x and y representing two consecutive terms in the series. Solving as a quadratic in x, we have $x=(2y+1)/2\pm\sqrt{(8y+1)/4}$. Hence $8y+1$ must be a square.

When $y=1$, $8y+1=3^2$, $x=3$;
$y=3$, $8y+1=5^2$, $x=6$;
$y=6$, $8y+1=7^2$, $x=10$;

and the series is, 1, 3, 6, 10, 15, 21, 28, 36, 45, etc., or the system of *triangular* numbers as set forth in Pascal's Triangle.

Also solved by *A. H. HOLMES*, *E. W. MORRELL*, *H. C. WILKES*, and *G. B. M. ZERR*.

44. Proposed by A. H. HOLMES, Box 963, Brunswick, Maine.

The hypotenuse of a right-angled triangle ABC, right-angled at A, is extended equally at both extremities so that $BE=CD$. Draw AD and AE. Find integral values for all the lines in the figure thus made.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Construct the figure as indicated by the problem. Then draw BF equal and parallel to AC, and draw CF, AF, EF, and DF. Then will $ABFC$ be a rectangle; and the diagonals BC and AF are equal.

It is also evident that $AE=DF$ and $AD=EF$. Whence $AEFD$ is an ob-